25 April 2005

By Hand

The Hon. Lau Kong-wah, JP
Chairman
Panel on Transport
c/o Legislative Council Secretariat
Legislative Council Building
8 Jackson Road, Central
Hong Kong

Dear Mr. Lau:

At the meeting of the LegCo Transport Panel on Friday I agreed to provide further information on the way the internal rate of return on equity is calculated and provide a summary of the NHKTC cash flow that would explain how it is used. A summary of the cashflow of the Company for the period from 1985 to 2003 is attached to match the figure of 8.4% which Dr So questioned in paragraph (1) of his letter.

The methodology of internal rate of return on equity is a standard method of assessing the return on an investment, both in the planning stage and in its subsequent evaluation[1]. It is the methodology agreed by the government and the NHKTC, and all their advisors in both arbitrations as the appropriate yardstick for measuring the reasonable remuneration over the life of the NHKTC franchise. Moreover the LegCo briefs for the Western Harbour Crossing Bill and the Route 3 Tunnel Bill make it clear that their toll adjustment mechanisms were based upon what was considered to be an appropriate IRR on equity over the respective franchises.

Dr So's points are based on what *Brealey and Meyers* in their standard work refer to as "the book rate of return" about which they say "the merits of an investment project do not depend on how accountants classify the cash flows and few companies these days make investment decisions just on the basis of book rate of return" (page 94).

[1] The experts in the arbitration referred to the standard work in the book The Principles of Corporate Finance by Brealey and Meyers. An extract pp. 93-105 attached for reference and the book itself can be produced if required.

The attached schedule shows the annual cash flow from 1985 to 2003 agreed by the experts on both sides. The internal rate of return on equity of the cashflow up to 2003 was 8.4% as can be easily verified using a spreadsheet program such as Excel. These calculations, and all others presented to the arbitrators were examined and agreed as correct by the financial experts representing both the New Hong Kong Tunnel Co. Ltd. and the Government, and they based their evidence on the agreed figures. The experts were:

- Mr Kenneth G. Morrison, senior partner of Moores Rowland Mazars, Certified Public Accountants (representing the NHKTC)
- Mr Macleod of Deutsche Bank AG (representing the government)

Last week Mr. Kenneth G. Morrison was asked to comment on the analysis in Dr So's 20 April letter and replied as follows:

> "*One of the main shortcomings of Dr So's analysis is that he has not factored into his calculation the time value of money. When looking at returns on long-term capital projects, IRR is frequently the method adopted for examining returns over the life of such project. This necessitates looking at cashflows paid and received throughout the project as well as the timing of such flows. Simply put, HK$1 paid/received in 1986 is not equivalent to HK$1 paid or received in 2005. Therefore, a capital sum of HK$750 million – the capital of EHT – in 1986 would be equivalent today to a considerably higher sum than HK$750 million. To measure returns achieved today against a capital sum in 1986 without adjusting for the time value of money is misleading and significantly distorts the true returns actually achieved.*
>
> *One of the main reasons for EHT maintaining a relatively high level of retained earnings in the accounts is because the consortium behind EHT had been restricted by the loan agreement with the lending banks for payment of dividends from retained earnings until such bank loans had been repaid in full. It is only in recent years that this has occurred. Furthermore, after the repayment of the bank loans in 2001, EHT did not have available cash funds to finance payments of additional dividends from retained earnings.*"

This independent analysis was confirmed by the NHKTC's own Financial Controller whose observations are:

1. *Dr So has taken the sum of the profits after tax of the Company for the years up to 2003 and average it to calculate the average return on equity.*

   This method simply has not taken into account the time value of money which is essential in any type of investment analysis. To illustrate, let us take an example of two projects:

   (i) *Project A costs $100 million at the beginning of year 1 and gives the investor a return of $10 million each year for the* *next 30 years (a total of $300 million) and returns the $100 million to the investor at the end of 30 year.*

(ii) Project B costs the same but gives the investor a return of $300 million and returns the $100 million to the investor, <u>both</u> at the end of 30 years.

Using Dr So's methodology, the average return for project A and project B will be the same and is equal to 10% (= the total return of $300 million ÷ $100 million ÷ 30 years). If we take into account the time value of money by calculating the internal rate of return, project A returns 10% whilst project B returns only 4.73%.

2. *An internal rate of return on equity for a project is the discount rate whereby the present value of the initial investment of the shareholders (cash paid out) will equal to the present value of the cash inflow to shareholders over the life of the project.*

   Dr So used the profit after tax for a particular year (2003) to say that the Company is making 33% return a year. It is wrong because (1) this method is not using the cashflow to calculate an IRR; (2) it has ignored the fact that the cashflow to shareholders in the first 9 years of the project was zero.

3. *Dr So said that the Company has retained its profit without distributing to shareholders so as to make use of finance to lower its return. This is not the case. The Company has actually distributed all its surplus cash to shareholders after it repaid its loan in 2001. This can be easily read from the Company's Balance Sheet, the cash on hand as at the end of 2003 was $42 million (a balance awaiting periodic distribution) while the retained earnings were $460 million. The Company simply did not have any extra cash to pay more dividends because most of the cash earned in the first 15 years of the project was used for repaying the loan that partially financed the building of the tunnel. The shareholders' equity plus retained earnings represent the long-term fixed assets held by the Company i.e. the Tunnel itself, which will be written down to zero by 2016 when the tunnel has to be returned to the Government with no payment in return. Using Dr So's methodology his return on equity would rise over the next 10 years merely because the retained earnings will reduce to reflect the value of the assets they represent reduces because they have to be returned to the government.*

We believe we have already addressed the queries of Dr So. We would like to point out that in the two arbitrations in 1997 and 2004, four separate financial experts plus the arbitrators involved accepted that an IRR on equity after tax is the appropriate way of measuring reasonable but not excessive remuneration over the 30-year franchise.

Regarding the meeting of the LegCo Subcommittee to study the Eastern Harbour Crossing Ordinance (Amendment of Schedule) Notice 2005 scheduled for 26 April

2005, please be informed that Ms Becky Fung, our Project Manager, will represent the Company to attend the meeting and answer any remaining questions that may arise on our accounts.

With best regards,

Yours sincerely,

Vernon F. Moore
Chairman

Attachment

cc : Mr. Andy Lau, Clerk to Subcommittee
Dr. Sarah Liao Sau-tung, Secretary for the Environment, Transport and Works
Secretary for the Environment, Transport and Works (Attn : Mr. Clement Lau)

NEW HONG KONG TUNNEL CO. LTD.
CALCULATION OF IRR ON EQUITY
HK$' 000

YEAR	CASH INVESTED	DIVIDEND TO SHAREHOLDERS	NET FLOW	PROFIT FOR THE YEAR
1986	-200,000	0	-200,000	0
1987	-200,000	0	-200,000	0
1988	-200,000	0	-200,000	0
1989	-150,000	0	-150,000	-33,746
1990	0	0	0	-117,449
1991	0	0	0	-22,217
1992	0	0	0	94,901
1993	0	0	0	142,448
1994	0	162,750	162,750	144,018
1995	0	119,250	119,250	117,738
1996	0	132,000	132,000	141,765
1997	0	137,250	137,250	150,113
1998	0	217,500	217,500	268,744
1999	0	123,750	123,750	304,101
2000	0	90,000	90,000	317,931
2001	0	240,000	240,000	320,906
2002	0	381,000	381,000	306,234
2003	0	327,000	327,000	255,059
	-750,000	1,930,500	1,180,500	2,390,546
		IRR up to 2003	**8.4%**	

"But which financial assets?" Vegetron's CFO queries. "The fact that investors expect only 12 percent on IBM stock does not mean that we should purchase Fly-by-Night Electronics if it offers 13 percent."

Your reply: "The opportunity-cost concept makes sense only if assets of equivalent risk are compared. In general, you should identify financial assets with risks equivalent to the project under consideration, estimate the expected rate of return on these assets, and use this rate as the opportunity cost."

Net Present Value's Competitors

Let us hope that the CFO is by now convinced of the correctness of the net present value rule. But it is possible that the CFO has also heard of some alternative investment criteria and would like to know why you do not recommend any of them. Just so that you are prepared, we will now look at three of the alternatives. They are:

1. The book rate of return.
2. The payback period.
3. The internal rate of return.

Later in the chapter we shall come across one further investment criterion, the profitability index. There are circumstances in which this measure has some special advantages.

Three Points to Remember about NPV

As we look at these alternative criteria, it is worth keeping in mind the following key features of the net present value rule. First, the NPV rule recognizes that *a dollar today is worth more than a dollar tomorrow,* because the dollar today can be invested to start earning interest immediately. Any investment rule which does not recognize the *time value of money* cannot be sensible. Second, net present value depends solely on the *forecasted cash flows* from the project and the *opportunity cost of capital.* Any investment rule which is affected by the manager's tastes, the company's choice of accounting method, the profitability of the company's existing business, or the profitability of other independent projects will lead to inferior decisions. Third, *because present values are all measured in today's dollars, you can add them up.* Therefore, if you have two projects A and B, the net present value of the combined investment is

$$\text{NPV}(A + B) = \text{NPV}(A) + \text{NPV}(B)$$

This additivity property has important implications. Suppose project B has a negative NPV. If you tack it onto project A, the joint project (A + B) will have a lower NPV than A on its own. Therefore, you are unlikely to be misled into accepting a poor project (B) just because it is packaged with a good one (A). As we shall see, the alternative measures do not have this additivity property. If you are not careful, you may be tricked into deciding that a package of a good and a bad project is better than the good project on its own.

NPV Depends on Cash Flow, Not Accounting Income

Net present value depends only on the project's cash flows and the opportunity cost of capital. But when companies report to shareholders, they do not simply show the cash flows. They also report book—that is, accounting—income and book assets; book income gets most of the immediate attention.

Financial managers sometimes use these numbers to calculate a book rate of return on a proposed investment. In other words, they look at the prospective book income as a proportion of the book value of the assets that the firm is proposing to acquire:

$$\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}$$

Cash flows and book income are often very different. For example, the accountant labels some cash outflows as *capital investments* and others as *operating expenses*. The operating expenses are, of course, deducted immediately from each year's income. The capital expenditures are put on the firm's balance sheet and then depreciated according to an arbitrary schedule chosen by the accountant. The annual depreciation charge is deducted from each year's income. Thus the book rate of return depends on which items the accountant chooses to treat as capital investments and how rapidly they are depreciated.[1]

Now the merits of an investment project do not depend on how accountants classify the cash flows[2] and few companies these days make investment decisions just on the basis of the book rate of return. But managers know that the company's shareholders pay considerable attention to book measures of profitability and naturally, therefore, they think (and worry) about how major projects would affect the company's book return. Those projects that will reduce the company's book return may be scrutinized more carefully by senior management.

You can see the dangers here. The book rate of return may not be a good measure of true profitability. It is also an *average* across all of the firm's activities. The average profitability of past investments is not usually the right hurdle for new investments. Think of a firm that has been exceptionally lucky and successful. Say its average book return is 24 percent, double shareholders' 12 percent opportunity cost of capital. Should it demand that all *new* investments offer 24 percent or better? Clearly not: That would mean passing up many positive-NPV opportunities with rates of return between 12 and 24 percent.

We will come back to the book rate of return in Chapter 12, when we look more closely at accounting measures of financial performance.

5.2 PAYBACK

Some companies require that the initial outlay on any project should be recoverable within a specified period. The **payback** period of a project is found by counting the number of years it takes before the cumulative forecasted cash flow equals the initial investment.

[1] This chapter's mini-case contains simple illustrations of how book rates of return are calculated and of the difference between accounting income and project cash flow. Read the case if you wish to refresh your understanding of these topics. Better still, do the case calculations.

[2] Of course, the depreciation method used for tax purposes does have cash consequences which should be taken into account in calculating NPV. We cover depreciation and taxes in the next chapter.

Consider the following three projects:

	Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	Payback Period (years)	NPV at 10%
A	−2,000	500	500	5,000	3	+2,624
B	−2,000	500	1,800	0	2	−58
C	−2,000	1,800	500	0	2	+50

Project A involves an initial investment of $2,000 ($C_0 = -2{,}000$) followed by cash inflows during the next three years. Suppose the opportunity cost of capital is 10 percent Then project A has an NPV of +$2,624:

$$\text{NPV(A)} = -2{,}000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{5{,}000}{1.10^3} = +\$2{,}624$$

Project B also requires an initial investment of $2,000 but produces a cash inflow of $500 in year 1 and $1,800 in year 2. At a 10 percent opportunity cost of capital project B has an NPV of −$58:

$$\text{NPV(B)} = -2{,}000 + \frac{500}{1.10} + \frac{1{,}800}{1.10^2} = -\$58$$

The third project, C, involves the same initial outlay as the other two projects but its first-period cash flow is larger. It has an NPV of +$50.

$$\text{NPV(C)} = -2{,}000 + \frac{1{,}800}{1.10} + \frac{500}{1.10^2} = +\$50$$

The net present value rule tells us to accept projects A and C but to reject project B.

The Payback Rule

Now look at how rapidly each project pays back its initial investment. With project A you take three years to recover the $2,000 investment; with projects B and C you take only two years If the firm used the payback *rule* with a cutoff period of two years, it would accept only projects B and C; if it used the payback rule with a cutoff period of three or more years, it would accept all three projects Therefore, regardless of the choice of cutoff period, the payback rule gives answers different from the net present value rule.

You can see why payback can give misleading answers:

1. *The payback rule ignores all cash flows after the cutoff date.* If the cutoff date is two years, the payback rule rejects project A regardless of the size of the cash inflow in year 3.
2. *The payback rule gives equal weight to all cash flows before the cutoff date.* The payback rule says that projects B and C are equally attractive, but, because C's cash inflows occur earlier, C has the higher net present value at any discount rate.

In order to use the payback rule, a firm has to decide on an appropriate cutoff date. If it uses the same cutoff regardless of project life, it will tend to accept many poor short-lived projects and reject many good long-lived ones.

Some companies discount the cash flows before they compute the payback period. The **discounted-payback rule** asks, How many periods does the project have to last in order to make sense in terms of net present value? This modification to the payback rule surmounts the objection that equal weight is given to all flows before the cutoff date. However, the discounted-payback rule still takes no account of any cash flows after the cutoff date.

5.3 INTERNAL (OR DISCOUNTED-CASH-FLOW) RATE OF RETURN

Whereas payback and return on book are ad hoc measures, internal rate of return has a much more respectable ancestry and is recommended in many finance texts. If, therefore, we dwell more on its deficiencies, it is not because they are more numerous but because they are less obvious.

In Chapter 2 we noted that net present value could also be expressed in terms of rate of return, which would lead to the following rule: "Accept investment opportunities offering rates of return in excess of their opportunity costs of capital." That statement, properly interpreted, is absolutely correct. However, interpretation is not always easy for long-lived investment projects.

There is no ambiguity in defining the true rate of return of an investment that generates a single payoff after one period:

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1$$

Alternatively, we could write down the NPV of the investment and find that discount rate which makes NPV = 0.

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{discount rate}} = 0$$

implies

$$\text{Discount rate} = \frac{C_1}{-C_0} - 1$$

Of course C_1 is the payoff and $-C_0$ is the required investment, and so our two equations say exactly the same thing. *The discount rate that makes NPV = 0 is also the rate of return.*

Unfortunately, there is no wholly satisfactory way of defining the true rate of return of a long-lived asset. The best available concept is the so-called **discounted-cash-flow (DCF) rate of return** or **internal rate of return (IRR).** The internal rate of return is used frequently in finance. It can be a handy measure, but, as we shall see, it can also be a misleading measure. You should, therefore, know how to calculate it and how to use it properly.

The internal rate of return is defined as the rate of discount which makes NPV = 0. This means that to find the IRR for an investment project lasting T years, we must solve for IRR in the following expression:

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \cdots + \frac{C_T}{(1 + \text{IRR})^T} = 0$$

Actual calculation of IRR usually involves trial and error. For example, consider a project that produces the following flows:

Cash Flows ($)		
C_0	C_1	C_2
−4,000	+2,000	+4,000

The internal rate of return is IRR in the equation

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1 + \text{IRR}} + \frac{4{,}000}{(1 + \text{IRR})^2} = 0$$

Let us arbitrarily try a zero discount rate. In this case NPV is not zero but +$2,000:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.0} + \frac{4{,}000}{(1.0)^2} = +\$2{,}000$$

The NPV is positive; therefore, the IRR must be greater than zero. The next step might be to try a discount rate of 50 percent. In this case net present value is −$889:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.50} + \frac{4{,}000}{(1.50)^2} = -\$889$$

The NPV is negative; therefore, the IRR must be less than 50 percent. In Figure 5.2 we have plotted the net present values implied by a range of discount rates. From this we can see that a discount rate of 28 percent gives the desired net present value of zero. Therefore IRR is 28 percent.

The easiest way to calculate IRR, if you have to do it by hand, is to plot three or four combinations of NPV and discount rate on a graph like Figure 5.2, connect the

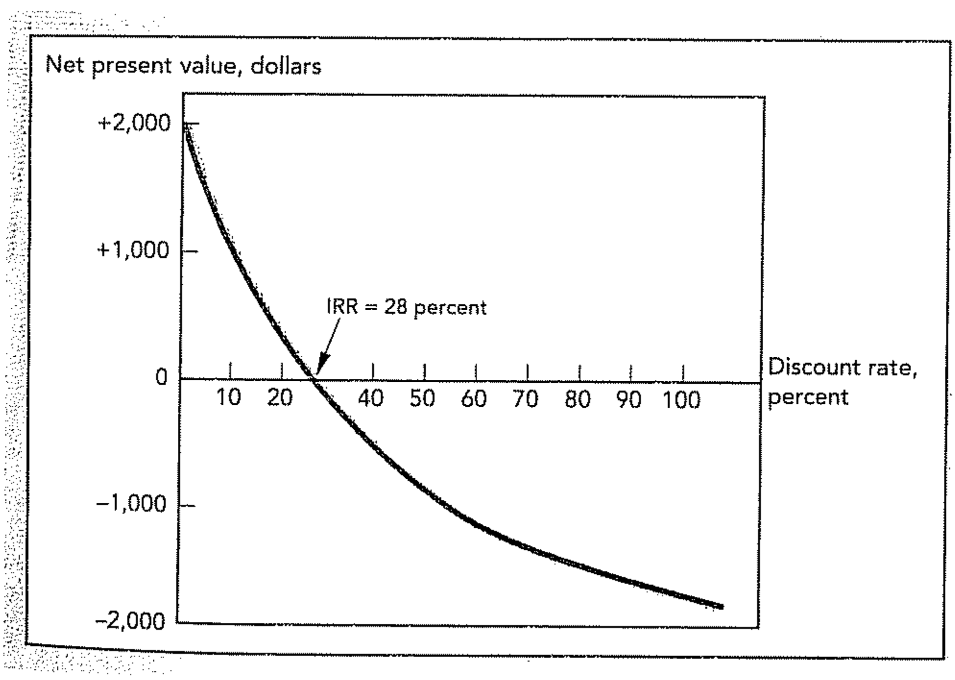


FIGURE 5.2

This project costs $4,000 and then produces cash inflows of $2,000 in year 1 and $4,000 in year 2. Its internal rate of return (IRR) is 28 percent, the rate of discount at which NPV is zero.

points with a smooth line, and read off the discount rate at which NPV = 0. It is of course quicker and more accurate to use a computer or a specially programmed calculator, and this is what most financial managers do.

Now, the *internal rate of return rule* is to accept an investment project if the opportunity cost of capital is less than the internal rate of return. You can see the reasoning behind this idea if you look again at Figure 5.2. If the opportunity cost of capital is less than the 28 percent IRR, then the project has a *positive* NPV when discounted at the opportunity cost of capital. If it is equal to the IRR, the project has a *zero* NPV. And if it is greater than the IRR, the project has a *negative* NPV. Therefore, when we compare the opportunity cost of capital with the IRR on our project, we are effectively asking whether our project has a positive NPV. This is true not only for our example. The rule will give the same answer as the net present value rule *whenever the NPV of a project is a smoothly declining function of the discount rate.*[3]

Many firms use internal rate of return as a criterion in preference to net present value. We think that this is a pity. Although, properly stated, the two criteria are formally equivalent, the internal rate of return rule contains several pitfalls.

Pitfall 1—Lending or Borrowing?

Not all cash-flow streams have NPVs that decline as the discount rate increases. Consider the following projects A and B:

Project	Cash Flows ($) C_0	C_1	IRR	NPV at 10%
A	−1,000	+1,500	+50%	+364
B	+1,000	−1,500	+50%	−364

Each project has an IRR of 50 percent. (In other words, $-1{,}000 + 1{,}500/1.50 = 0$ *and* $+1{,}000 - 1{,}500/1.50 = 0$.)

Does this mean that they are equally attractive? Clearly not, for in the case of A, where we are initially paying out $1,000, we are *lending* money at 50 percent; in the case of B, where we are initially receiving $1,000, we are *borrowing* money at 50 percent. When we lend money, we want a *high* rate of return; when we borrow money, we want a *low* rate of return.

If you plot a graph like Figure 5.2 for project B, you will find that NPV increases as the discount rate increases. Obviously the internal rate of return rule, as we stated it above, won't work in this case; we have to look for an IRR *less* than the opportunity cost of capital.

This is straightforward enough, but now look at project C:

Project	Cash Flows ($) C_0	C_1	C_2	C_3	IRR	NPV at 10%
C	+1,000	−3,600	+4,320	−1,728	+20%	−.75

[3] Here is a word of caution: Some people confuse the internal rate of return and the opportunity cost of capital because both appear as discount rates in the NPV formula. The internal rate of return is a *profitability measure* that depends solely on the amount and timing of the project cash flows. The opportunity cost of capital is a *standard of profitability* for the project which we use to calculate how much the project is worth. The opportunity cost of capital is established in capital markets. It is the expected rate of return offered by other assets equivalent in risk to the project being evaluated.

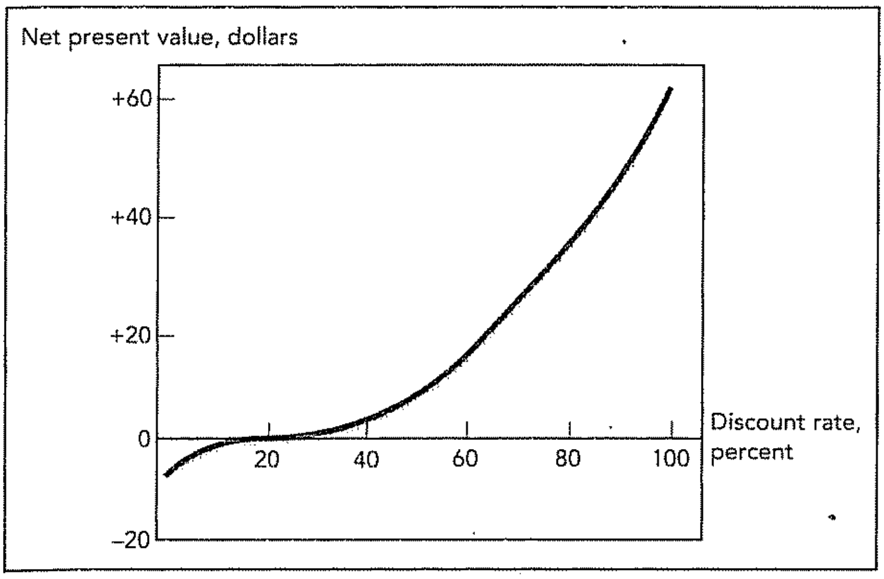


FIGURE 5.3

The NPV of project C increases as the discount rate increases.

It turns out that project C has zero NPV at a 20 percent discount rate. If the opportunity cost of capital is 10 percent, that means the project is a good one. Or does it? In part, project C is like borrowing money, because we receive money now and pay it out in the first period; it is also partly like lending money because we pay out money in period 1 and recover it in period 2. Should we accept or reject? The only way to find the answer is to look at the net present value. Figure 5.3 shows that the NPV of our project *increases* as the discount rate increases. If the opportunity cost of capital is 10 percent (i.e., less than the IRR), the project has a very small negative NPV and we should reject.

Pitfall 2—Multiple Rates of Return

In most countries there is usually a short delay between the time when a company receives income and the time it pays tax on the income. Consider the case of Albert Vore, who needs to assess a proposed advertising campaign by the vegetable canning company of which he is financial manager. The campaign involves an initial outlay of $1 million but is expected to increase pretax profits by $300,000 in each of the next five periods. The tax rate is 50 percent, and taxes are paid with a delay of one period. Thus the expected cash flows from the investment are as follows:

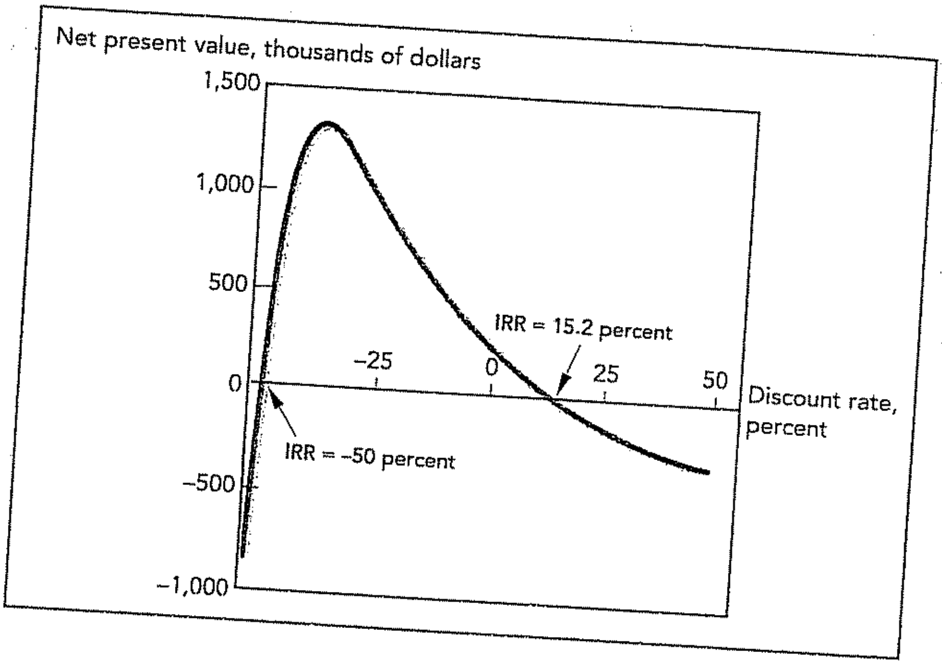
Cash Flows ($ thousands)

	Period						
	0	1	2	3	4	5	6
Pretax flow	−1,000	+300	+300	+300	+300	+300	
Tax		+500	−150	−150	−150	−150	−150
Net flow	−1,000	+800	+150	+150	+150	+150	−150

Note: The $1 million outlay in period 0 *reduces* the company's taxes in period 1 by $500,000; thus we enter +500 in year 1.

FIGURE 5.4

The advertising campaign has two internal rates of return. NPV = 0 when the discount rate is −50 percent and when it is +15.2 percent.



Mr. Vore calculates the project's IRR and its NPV as follows:

IRR (%)	NPV at 10%
−50 *and* 15.2	74.9 or $74,900

Note that there are *two* discount rates that make NPV = 0. That is, *each* of the following statements holds:

$$\text{NPV} = -1{,}000 + \frac{800}{.50} + \frac{150}{(.50)^2} + \frac{150}{(.50)^3} + \frac{150}{(.50)^4} + \frac{150}{(.50)^5} - \frac{150}{(.50)^6} = 0$$

and

$$\text{NPV} = -1{,}000 + \frac{800}{1.152} + \frac{150}{(1.152)^2} + \frac{150}{(1.152)^3} + \frac{150}{(1.152)^4} + \frac{150}{(1.152)^5} - \frac{150}{(1.152)^6} = 0$$

In other words, the investment has an IRR of both −50 *and* 15.2 percent. Figure 5.4 shows how this comes about. As the discount rate increases, NPV initially rises and then declines. The reason for this is the double change in the sign of the cash-flow stream. There can be as many different internal rates of return for a project as there are changes in the sign of the cash flows.[4]

[4] By Descartes's "rule of signs" there can be as many different solutions to a polynomial as there are changes of sign. For a discussion of the problem of multiple rates of return, see J. H. Lorie and L. J. Savage, "Three Problems in Rationing Capital," *Journal of Business* 28 (October 1955), pp. 229–239; and E. Solomon, "The Arithmetic of Capital Budgeting," *Journal of Business* 29 (April 1956), pp. 124–129.

In our example the double change in sign was caused by a lag in tax payments, but this is not the only way that it can occur. For example, many projects involve substantial decommissioning costs. If you strip-mine coal, you may have to invest large sums to reclaim the land after the coal is mined. Thus a new mine creates an initial investment (negative cash flow up front), a series of positive cash flows, and an ending cash outflow for reclamation. The cash-flow stream changes sign twice, and mining companies typically see two IRRs.

As if this is not difficult enough, there are also cases in which *no* internal rate of return exists. For example, project D has a positive net present value at all discount rates:

	Cash Flows ($)				
Project	C_0	C_1	C_2	IRR (%)	NPV at 10%
D	+1,000	-3,000	+2,500	None	+339

A number of adaptations of the IRR rule have been devised for such cases. Not only are they inadequate, but they also are unnecessary, for the simple solution is to use net present value.[5]

Pitfall 3—Mutually Exclusive Projects

Firms often have to choose from among several alternative ways of doing the same job or using the same facility. In other words, they need to choose from among **mutually exclusive projects.** Here too the IRR rule can be misleading.

Consider projects E and F:

	Cash Flows ($)			
Project	C_0	C_1	IRR (%)	NPV at 10%
E	-10,000	+20,000	100	+ 8,182
F	-20,000	+35,000	75	+11,818

[5]Companies sometimes get around the problem of multiple rates of return by discounting the later cash flows back at the cost of capital until there remains only one change in the sign of the cash flows. A *modified internal rate of return* can then be calculated on this revised series. In our example, the modified IRR is calculated as follows:

1. Calculate the present value of the year 6 cash flow in year 5:

$$\text{PV in year 5} = -150/1.10 = -136.36$$

2. Add to the year 5 cash flow the present value of subsequent cash flows:

$$C_5 + \text{PV(subsequent cash flows)} = 150 - 136.36 = 13.64$$

3. Since there is now only one change in the sign of the cash flows, the revised series has a unique rate of return, which is 15 percent:

$$\text{NPV} = -1{,}000 + \frac{800}{1.15} + \frac{150}{1.15^2} + \frac{150}{1.15^3} + \frac{150}{1.15^4} + \frac{13.64}{1.15^5} = 0$$

Since the modified IRR of 15 percent is greater than the cost of capital (and the initial cash flow is negative), the project has a positive NPV when valued at the cost of capital.

Of course, it would be much easier in such cases to abandon the IRR rule and just calculate project NPV.

Perhaps project E is a manually controlled machine tool and project F is the same tool with the addition of computer control. Both are good investments, but F has the higher NPV and is, therefore, better. However, the IRR rule seems to indicate that if you have to choose, you should go for E since it has the higher IRR. If you follow the IRR rule, you have the satisfaction of earning a 100 percent rate of return; if you follow the NPV rule, you are $11,818 richer.

You can salvage the IRR rule in these cases by looking at the internal rate of return on the incremental flows. Here is how to do it: First, consider the smaller project (E in our example). It has an IRR of 100 percent, which is well in excess of the 10 percent opportunity cost of capital. You know, therefore, that E is acceptable. You now ask yourself whether it is worth making the additional $10,000 investment in F. The incremental flows from undertaking F rather than E are as follows:

	Cash Flows ($)			
Project	C_0	C_1	IRR (%)	NPV at 10%
F–E	−10,000	+15,000	50	+3,636

The IRR on the incremental investment is 50 percent, which is also well in excess of the 10 percent opportunity cost of capital. So you should prefer project F to project E.[6]

Unless you look at the incremental expenditure, IRR is unreliable in ranking projects of different scale. It is also unreliable in ranking projects which offer different patterns of cash flow over time. For example, suppose the firm can take project G *or* project H but not both (ignore I for the moment):

	Cash Flows ($)								
Project	C_0	C_1	C_2	C_3	C_4	C_5	Etc.	IRR (%)	NPV at 10%
G	−9,000	+6,000	+5,000	+4,000	0	0		33	3,592
H	−9,000	+1,800	+1,800	+1,800	+1,800	+1,800	. . .	20	9,000
I		−6,000	+1,200	+1,200	+1,200	+1,200	. . .	20	6.000

Project G has a higher IRR, but project H has the higher NPV. Figure 5.5 shows why the two rules give different answers. The blue line gives the net present value of project G at different rates of discount. Since a discount rate of 33 percent produces a net present value of zero, this is the internal rate of return for project G. Similarly, the burgundy line shows the net present value of project H at different discount rates. The IRR of project H is 20 percent. (We assume project H's cash flows continue indefinitely.) Note that project H has a higher NPV so long as the opportunity cost of capital is less than 15.6 percent.

The reason that IRR is misleading is that the total cash inflow of project H is larger but tends to occur later. Therefore, when the discount rate is low, H has the higher NPV; when the discount rate is high, G has the higher NPV. (You can see from Figure 5.5 that the two projects have the *same* NPV when the discount rate is 15.6 percent.) The internal rates of return on the two projects tell us that at a discount rate of 20 percent H has a zero NPV (IRR = 20 percent) and G has a positive

[6] You may, however, find that you have jumped out of the frying pan into the fire. The series of incremental cash flows may involve several changes in sign. In this case there are likely to be multiple IRRs and you will be forced to use the NPV rule after all.

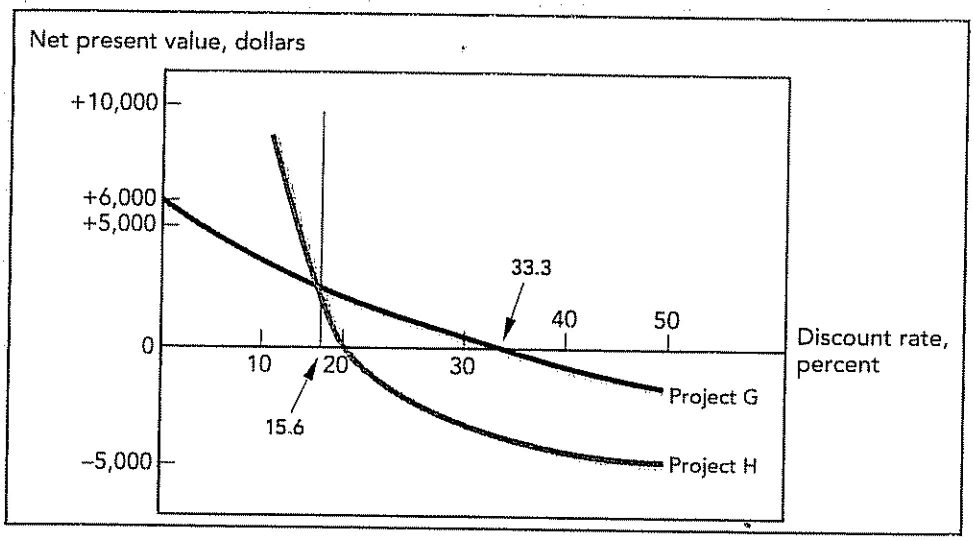


FIGURE 5.5

The IRR of project G exceeds that of project H, but the NPV of project G is higher *only* if the discount rate is greater than 15.6 percent.

NPV. Thus if the opportunity cost of capital were 20 percent, investors would place a higher value on the shorter-lived project G. But in our example the opportunity cost of capital is not 20 percent but 10 percent. Investors are prepared to pay relatively high prices for longer-lived securities, and so they will pay a relatively high price for the longer-lived project. At a 10 percent cost of capital, an investment in H has an NPV of $9,000 and an investment in G has an NPV of only $3,592.[7]

This is a favorite example of ours. We have gotten many businesspeople's reaction to it. When asked to choose between G and H, many choose G. The reason seems to be the rapid payback generated by project G. In other words, they believe that if they take G, they will also be able to take a later project like I (note that I can be financed using the cash flows from G), whereas if they take H, they won't have money enough for I. In other words they implicitly assume that it is a *shortage of capital* which forces the choice between G and H. When this implicit assumption is brought out, they usually admit that H is better if there is no capital shortage.

But the introduction of capital constraints raises two further questions. The first stems from the fact that most of the executives preferring G to H work for firms that would have no difficulty raising more capital. Why would a manager at IBM, say, choose G on the grounds of limited capital? IBM can raise plenty of capital and can take project I regardless of whether G or H is chosen; therefore I should not affect the choice between G and H. The answer seems to be that large firms usually impose capital budgets on divisions and subdivisions as a part of the firm's planning and control system. Since the system is complicated and cumbersome, the

[7]It is often suggested that the choice between the net present value rule and the internal rate of return rule should depend on the probable reinvestment rate. This is wrong. The prospective return on another *independent* investment should *never* be allowed to influence the investment decision. For a discussion of the reinvestment assumption see A. A. Alchian, "The Rate of Interest, Fisher's Rate of Return over Cost and Keynes' Internal Rate of Return," *American Economic Review* 45 (December 1955), pp. 938–942.

budgets are not easily altered, and so they are perceived as real constraints by dle management.

The second question is this. If there is a capital constraint, either real or imposed, should IRR be used to rank projects? The answer is no. The problem this case is to find that package of investment projects which satisfies the cap constraint and has the largest net present value. The IRR rule will not identify package. As we will show in the next section, the only practical and general w to do so is to use the technique of linear programming.

When we have to choose between projects G and H, it is easiest to compare th net present values. But if your heart is set on the IRR rule, you can use it as long you look at the internal rate of return on the incremental flows. The procedure exactly the same as we showed above. First, you check that project G has a satis factory IRR. Then you look at the return on the additional investment in H.

Project	Cash Flows ($)							IRR (%)	NPV at 10%
	C_0	C_1	C_2	C_3	C_4	C_5	Etc.		
H–G	0	−4,200	−3.200	−2,200	+1,800	+1,800		15.6	+5,408

The IRR on the incremental investment in H is 15.6 percent. Since this is greater than the opportunity cost of capital, you should undertake H rather than G.

Pitfall 4—What Happens When We Can't Finesse the Term Structure of Interest Rates?

We have simplified our discussion of capital budgeting by assuming that the opportunity cost of capital is the same for all the cash flows, C_1, C_2, C_3, etc. This is not the right place to discuss the term structure of interest rates, but we must point out certain problems with the IRR rule that crop up when short-term interest rates are different from long-term rates.

Remember our most general formula for calculating net present value:

$$\text{NPV} = C_0 + \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \frac{C_3}{(1 + r_3)^3} + \cdots$$

In other words, we discount C_1 at the opportunity cost of capital for one year, C_2 at the opportunity cost of capital for two years, and so on. The IRR rule tells us to accept a project if the IRR is greater than the opportunity cost of capital. But what do we do when we have several opportunity costs? Do we compare IRR with $r_1, r_2, r_3, \ldots$? Actually we would have to compute a complex weighted average of these rates to obtain a number comparable to IRR.

What does this mean for capital budgeting? It means trouble for the IRR rule whenever the term structure of interest rates becomes important.[8] In a situation where it is important, we have to compare the project IRR with the expected IRR (yield to maturity) offered by a traded security that (1) is equivalent in risk to the project and (2) offers the same time pattern of cash flows as the project. Such a comparison is easier said than done. It is much better to forget about IRR and just calculate NPV.

[8] The source of the difficulty is that the IRR is a derived figure without any simple economic interpretation. If we wish to define it, we can do no more than say that it is the discount rate which applied to all cash flows makes NPV = 0. The problem here is not that the IRR is a nuisance to calculate but that it is not a useful number to have.

Many firms use the IRR, thereby implicitly assuming that there is no difference between short-term and long-term rates of interest. They do this for the same reason that we have so far finessed the term structure: simplicity.[9]

The Verdict on IRR

We have given four examples of things that can go wrong with IRR. We spent much less space on payback or return on book. Does this mean that IRR is worse than the other two measures? Quite the contrary. There is little point in dwelling on the deficiencies of payback or return on book. They are clearly ad hoc measures which often lead to silly conclusions. The IRR rule has a much more respectable ancestry. It is a less easy rule to use than NPV, but, used properly, it gives the same answer.

Nowadays few large corporations use the payback period or return on book as their primary measure of project attractiveness. Most use discounted cash flow or "DCF," and for many companies DCF means IRR, not NPV. We find this puzzling, but it seems that IRR is easier to explain to nonfinancial managers, who think they know what it means to say that "Project G has a 33 percent return." But can these managers use IRR properly? We worry particularly about Pitfall 3. The financial manager never sees all possible projects. Most projects are proposed by operating managers. Will the operating managers' proposals have the highest NPVs or the highest IRRs?

A company that instructs nonfinancial managers to look first at projects' IRRs prompts a search for high-IRR projects. It also encourages the managers to *modify* projects so that their IRRs are higher. Where do you typically find the highest IRRs? In short-lived projects requiring relatively little up-front investment. Such projects may not add much to the value of the firm.

5.4 CHOOSING CAPITAL INVESTMENTS WHEN RESOURCES ARE LIMITED

Our entire discussion of methods of capital budgeting has rested on the proposition that the wealth of a firm's shareholders is highest if the firm accepts *every* project that has a positive net present value. Suppose, however, that there are limitations on the investment program that prevent the company from undertaking all such projects. Economists call this *capital rationing*. When capital is rationed, we need a method of selecting the package of projects that is within the company's resources yet gives the highest possible net present value.

An Easy Problem in Capital Rationing

Let us start with a simple example. The opportunity cost of capital is 10 percent, and our company has the following opportunities:

	Cash Flows ($ millions)			
Project	C_0	C_1	C_2	NPV at 10%
A	−10	+30	+5	21
B	−5	+5	+20	16
C	−5	+5	+15	12

[9] ... we will look at some other cases in which it would be misleading to use the same discount ... short-term and long-term cash flows.