INFORMATION NOTE

Proportional Representation Electoral Systems

1.   Meaning of Proportional Representation

1.1   Proportional representation (PR) is a generic term for all the systems of election which seek to relate seats to votes cast by the electorate in accordance with party or candidate preference.   The form can be multi-member seats or party lists.

1.2   The aim of PR is to have the allocation of seats in representative bodies according to the number of votes received by parties or by individual candidates.   We take one constituency with five seats as an example. The party polling three-fifths of the total votes cast by the voters in that constituency will return three out of five seats; the party polling two-fifths of the total votes will return two seats.   The party composition of the representative body will then reflect the party preference of the electorate with a considerable degree of accuracy in the constituency. Table 1 illustrates a representative body with five seats representing one constituency.

Table 1: Illustrative example of the representative body with five seats with 100,000 voters in one constituency

Party	Votes		Seats in representative body	
	Number cast	Percentage	Number returned	Percentage
A	60,000	60%	3	60%
B	40,000	40%	2	40%
TOTAL	100,000	100%	5	100%

1.3   Upon this simple foundation, however, some complicated structures are built. The complications arise from two distinct aims: (i) to relate as accurately as possible the number of seats held to the polling strength of the parties, and (ii) to permit an opportunity for the voter to express an opinion on the personal merits of the candidates.

2. Party List System

2.1   In the party list system, the voter may not need to know the names of the candidates for whom the voter is voting, because the system is based exclusively on party and its list of candidates. Thus, the voter can only support a party, and in the original form of this system the voter cannot support any individual candidates.

2.2   When the voter enters the voting booth, the voter is presented with a series of parties, for one of which the voter can cast the vote. The candidates of the party may be listed under the name of the party. Once all the ballots are cast, they are totalled, and each party receives the number of quotas in the representative body corresponding to the number of quotas awarded it by the popular vote.

2.3   Since more names are on the party lists than each party will win, the seats won are counted off beginning with the first name on the party list and proceeding down the list until all of that party's seats are assigned (see the illustrative example of Table 2). Obviously, the further down the list an individual candidate's name lies, the less chance the candidate has getting elected, and the higher up on the list the better chance the candidate has and probably the more important the candidate is to the party. In any case, the party decides the position to favour a particular person. Further, in the system of proportionality, the voter cannot express any direct feeling toward any candidate. Thus, the voter may sense some form of isolation from the candidates and the election system.

Table 2: Illustrative example of the voting result in the representative body with five seats with 100,000 voters in one constituency

Party	Votes		Seats in representative body	
	Number cast	Percentage	Number returned	Percentage
A	60,000	60%	1. John SMITH* 2. Mike BROWN* 3. Bob WHITE* [4. Rick BLACK] [5. Nick BAKER]	60%
B	40,000	40%	1. Mary JONES* 2. Rose CARTER* [3. Alice GRAY] [4. Peter BELL] [5. May LEE]	40%
TOTAL	100,000	100%	5	100%

Note 1: * represents elected candidate.
Note 2: [ ] represents name on the list but seat not awarded because of the low order in the party list.

3. Proportional Representation Formulas

3.1 PR formulas can be explained by classifying and subclassifying them: the first classification distinguishes between list PR, in which voters cast their votes for party lists of candidates, and the single transferable vote (STV), in which voters vote for individual candidates.

3.2 List PR can then be classified into largest remainders (quota) and highest averages (divisor) systems. And these can be classified further according to the particular quota or divisor that they employ. A few simple examples are used to illustrate their operation.

Largest Reminders Formulas

3.3 The spirit of a largest remainder (LR) minimizes wastage of votes cast. In practice, it enables PR because the remaining votes are counted in a quota system to allocate a seat. When the full quota results in a remaining seat, the party having the largest remainder would obtain the remaining seat.

3.4 In all quota systems, it is required to calculate a quota of votes that entitles parties to a seat; a party gets as many seats as it has quotas of votes; any unallocated seats are given to those parties having the largest numbers of unused votes (i.e. remainders).

3.5 The three most common LR formulas, using the Hare, Droop, and Imperiali quotas, are shown in Table 3.

- The Hare quota is the oldest and simplest of the quotas: it is simply the total number of valid votes divided by the number of seats at stake in a constituency. The denominator is the number of seats contested.

- The Droop quota divides the total number of votes by the number of seats plus 1. The denominator is the number of seats contested plus one.

- The Imperiali quota divides the total number of votes by the number of seats plus 2. The denominator is the number of seats contested plus two.

3.6 The procedure is to divide each party's votes by the quota, which yields the number of full plus remaining quotas each party has won. Parties then receive one seat for each full quota; any seats that cannot be allocated in this way are given to the parties with the largest fraction of a remaining quota (i.e. the largest remainder).

3.7　　　　　In the first example of Table 3, based on the use of the Hare quota, parties A, B, C, and D have 3, 2, 1, and 1 full Hare quotas respectively and are therefore given 3, 2, 1, and 1 seats -- a total of 7 seats -- in the initial allocation.

3.8　　　　　Since 8 seats are available, one more seat has to be allocated: it goes to the party with the largest remainder of votes, that is, the largest remaining fraction of a quota.　This is party C with a remaining 0.36 of a Hare quota; and the final distribution of seats becomes 3, 2, 2, 1.

3.9　　　　　The procedure for allocating seats to parties with the Droop and Imperiali methods is exactly the same, i.e. any remaining seats are allocated to the largest remainder.　However, it should be noted that by the use of Imperiali quota system the largest remainder is not able to capture any remaining seat because all seats have been allocated under the full quota calculation.

3.10　　　　Based on either the Droop quota or the Imperiali quota, the final distribution of seats to parties A (4 seats), B (2 seats), C (1 seat), and D (1 seat) has the same result.

Table 3: Illustrative examples of the operation of three largest remainders formulas in an eight-member district with four parties

Hare quota = 100,000 votes/8 seats = 12,500 votes = 1 seat

Party	Votes	Hare quotas	Full quota Seats	Remainder	Remaining seats	Total seats
A	41,000	3.28	3	0.28	0	3
B	29,000	2.32	2	0.32	0	2
C	17,000	1.36	1	0.36 [1]	1 [1]	2
D	13,000	1.04	1	0.04	0	1
TOTAL	100,000	8.00	7	--	1	8

Note 1: The largest remainder captures the one remaining seat.

Droop quota = 100,000 votes/(8 + 1)[2] seats = 11,111 votes = 1 seat

Party	Votes	Droop quotas	Full quote Seats	Remainder	Remaining seats	Total seats
A	41,000	3.69	3	0.69 [3]	1 [3]	4
B	29,000	2.61	2	0.61	0	2
C	17,000	1.53	1	0.53	0	1
D	13,000	1.17	1	0.17	0	1
TOTAL	100,000	9.00	7	--	1	8

Note 2: The Droop quota has the number of seats plus one as denominator.
Note 3: The largest remainder captures the one remaining seat.

Imperiali quota = 100,000 votes/(8 + 2)[4] seats = 10,000 votes = 1 seat

Party	Votes	Imperiali quotas	Full quota Seats	Remainder	Remaining seats	Total seats
A	41,000	4.10	4	0.10	0	4
B	29,000	2.90	2	0.90 [5]	0 [5]	2
C	17,000	1.70	1	0.70	0	1
D	13,000	1.30	1	0.30	0	1
TOTAL	100,000	10.00	8	--	0	8

Note 4: The Imperiali quota has the number of seats plus two as denominator.
Note 5: The largest remainder is not able to capture any remaining seat because all seats have been allocated under the full quota calculation.

3.11   Based on the examples of Table 3, the following observations can be made.

     (a)   Although the four parties obtain the same number of votes, the number of seats allocated to each party is different based on the method of calculating the quota.

     (b)   In terms of the value of a seat to be allocated, the three quota systems have different results. If a party wins 10,000 votes, it will obtain a full quota only in the Imperiali system (i.e. a seat for the party).

Table 4: Comparisons of Hare, Droop and Imperiali quota systems

Quota system	Denominator	Worth of one seat (number of votes)
Hare	Smallest (8)	Highest (12,500)
Droop	Medium (8 + 1)	Medium (11,111)
Imperiali	Largest (8 + 2)	Smallest (10,000)

     (c)   The larger the denominator, the more seats will a larger party capture. Party A is an example. It is allocated only three seats in the Hare quota system; under the Droop and Imperiali quota systems, party A is allocated four seats.

     (d)   The larger the denominator, the less significant will be the remainder; hence, the less likelihood of a small party has allocated the remaining seat. Party B is an example. In the Imperiali quota system, party B obtains the largest remainder of 0.90, but there is no remaining seat to be allocated. The maximum disadvantage for small party occurs when there is no remaining seat to be allocated at all.

     (e)   Among the quota systems, proportionality decreases as the quota decreases. The use of the Droop quota instead of the Hare quota causes the small party C to lose a seat and the largest Party A to win an extra seat.

Highest Averages Formulas

3.12   These are mathematically derived formulas which have been developed by experts in Europe in the hope of ensuring a more proportional result. Seats are awarded sequentially to parties having the highest 'average' numbers of votes per seat. The 'average' is the value of the quotient obtained from the application of a particular divisor system. Each time a party receives a seat, its 'average' goes down. Table 5 shows the two highest averages methods that are in common use for allocation of seats to parties: d'Hondt and modified Sainte-Laguë.

d'Hondt Formula

3.13   The d'Hondt formula uses the integers 1, 2, 3, 4, and so on as divisors. As Table 5 shows, the first seat (indicated by the number in parenthesis) goes to the largest party, party A, whose votes are then divided by 2. The second seat is given to party B, because its 'average' (29,000 votes, its original vote total) is higher than C's and D's and also higher than A's votes divided by 2. The third seat goes to A because its vote divided by 2 is higher than B's vote divided by 2 and higher than C's and D's votes; and so on. The final seat allocation to parties A, B, C, and D is 3, 2, 1, and 0 seats.

Sainte-Laguë Formula

3.14   The Sainte-Laguë formula, in the original form proposed by its inventor, uses the odd-integer divisor series 1, 3, 5, 7, and so on as divisors. In practice, it is used in a modified form which uses 1.4 instead of 1 as the first divisor. The use of 1.4 as a divisor ensures a more proportional result than the original form.

3.15   Its sequential procedure for allocating seats to parties is identical to that of the d'Hondt method. In the example of Table 5, the first five seats are awarded to the parties in exactly the same order as in the d'Hondt method, but the sixth seat is won by party D instead of A; the final distribution of seats therefore becomes 2, 2, 1, 1.

Table 5: Illustrative examples of the operation of two highest averages formulas in a six-member district with four parties

Seats allocated using d'Hondt divisors of 1, 2, 3, etc.

Party	Votes (V)	V/1	V/2	V/3	Total seats
A	41,000	41,000 (1)	20,500 (3)	13,667 (6)	3
B	29,000	29,000 (2)	14,500 (5)	9,667	2
C	17,000	17,000 (4)	8,500		1
D	13,000	13,000			0
TOTAL	100,000				6

Seats allocated using modified Sainte-Laguë divisors of 1.4, 3, 5, etc.

Party	Votes (V)	V/1.4	V/3	V/5	Total seats
A	41,000	29,286 (1)	13,667 (3)	8,200	2
B	29,000	20,714 (2)	9,667 (5)	5,800	2
C	17,000	12,143 (4)	5,667		1
D	13,000	9,286 (6)			1
TOTAL	100,000				6

Note 1: The order in which seats are awarded sequentially to parties is indicated by the numbers in parentheses.
Note 2: There is no need to employ a higher divisor once the seats can all be allocated.

3.16   The following observations can be made from examples of Tables 3 and 5.

(a) The examples of Tables 3 and 5 are selected not just to illustrate the different procedures but also to show that the choice of PR formula can affect the allocation of seats. Such differences do not always appear; in a seven-member district, for instance, the four parties would be awarded exactly the same 3, 2, 1, and 1 seats, respectively, by all five methods.

(b) Where differences do appear, however, they are not random but systematically affect the degree of proportionality and the electoral opportunities for small parties.

(c) By the use of modified Sainte-Laguë formula, party A loses one seat to party D. The use of modified Sainte-Laguë formula would benefit small party.

(d) All LR formulas suffer from the so-called Alabama paradox -- the possibility that when the total number of seats in a district is increased by one, one of the parties may actually lose a seat. In the illustrative four-party and LR-Hare system, this happens to party C when the district magnitude is increased from eight to nine: in the eight-member district it wins 2 seats, but only 1 in a nine-member district. Divisor systems are free from this defect because they award seat subsequently; hence adding a seat to the total of the available seats in a district cannot change the allocation of seats that have already been allocated to the parties in previous steps.

4.   Single Transferable Vote (STV)

Methodology of STV

4.1   By the use of STV, voters cast their votes for individual candidates, in order of the voters' preferences, instead of party lists. It is considered an complex device, both for the voter and for the returning officer, but it has the advantage of reducing party influence while maintaining the principle of proportionality.

4.2   In operation, a single list of candidates is presented to the voter, and the party affiliation of each candidate may or may not be included, since the workings of the system require only the name of the candidates and not their party. The voter ranks by number the candidates in order of the preference, with the most favoured candidate being number one, the next number two, and so on down the list.

4.3   The function of party in the voter's activity is to guide the voter's preferences for one party, the voter can select those candidates, giving them the top rankings. The voter can also ignore party guidelines entirely and merely indicate the preferences according to the feeling about each candidate on the list.

4.4   Like LR systems, STV requires the choice of a quota, which in practice is always the Droop quota. However, it is defined in a slightly different way from the LR Droop quota: the quotient arrived at by dividing the total vote by the number of seats plus 1 is rounded up or, if the quotient is an integer, 1 is added. In the example of Table 6, the LR Droop quota would be 25, but the STV Droop quota is 26.

4.5   Following the calculation of the quota the first preference votes for each candidate are counted and any candidate who achieves a number of first preference votes greater or equal to the quota is declared elected. The votes obtained by any such candidate in excess of the quota can be regarded as surplus votes and these are distributed in accordance with second preferences.

4.6   The case of which votes to distribute is handled by sorting all the votes for an elected candidate by the next preferences. The same proportion of votes for each next preference is distributed as the total surplus divided by the total transferable vote for the elected candidate.

4.7   The ballots which are transferred in this way are taken from the last ones added to each pile of ballots. Consequently in the later stages of the count the ballots involved are likely to be ones which themselves have already been transferred.

4.8   If no candidate has obtained a number of votes equal to or greater than the quota, the candidate with the lowest number of first preference votes is eliminated and the votes for that candidate are distributed in accordance with the second preferences indicated on the ballots.

4.9   The process of distributing surplus votes and eliminating candidates continues until the number of seats have been filled. Precedence is given in this process to distributing the surplus votes rather than eliminating a candidate if the surplus is greater than the difference in votes between the bottom two candidates. Otherwise, a candidate is eliminated before a surplus is distributed.

Illustrative Example

4.10   Table 6 presents an example that illustrates all of STV's basic rules. The counting procedure can be illustrated which sets out the result of each count.   In a three-member district, there are 100 voters and seven candidates (P to V). There are 41 ballots on which the voter has indicated the preferences for three candidates.

4.11   In the top half of the table, the voters' preferences are summarized: there are 15 ballots with candidate P as the first preference, Q as the second preference, and R in third place; 26 ballots also contain three preferences but in different rankings of candidates. There are 29 ballots with two preferences, and 30 ballots with only one preference.

4.12   In the first count, the ballots are arranged according to first preferences. If a candidate has a Droop quota or more of these first preferences, the candidate is elected: candidate P with 30 votes in the example of Table 6.

4.13   In the second count, P's 4 surplus votes are transferred to the next lower preferences, half to Q and half to R, because the original 30 ballots with P as first preference are also split equally between Q and R as second preferences.

4.14　　　　Since the second count does not yield another candidate with the Droop quota necessary for election, the weakest candidate R is eliminated and the 5 votes transferred to the next preference on the ballots candidate Q in the third count.

4.15　　　　This procedure has to be repeated in the fourth count with the elimination of candidate T and the transfer of the votes to candidate S -- who now exceeds the Droop quota and is elected.

4.16　　　　S's 3 surplus votes should be transferred to the next preference, but because no further preferences are indicated on the ballots, these votes become non-transferable.

4.17　　　　In the sixth count, the weakest candidate is again eliminated (candidate V); only four candidates are left, and candidate Q is next in line for elimination; this means that no further calculations are necessary and that candidate U is the third candidate to be elected.

4.18　　　　Hence, after all the counts, candidates P, S and U are elected. It should be noted that candidate U is elected without reaching quota.

Table 6: Illustrative example of the operation of the single transferable vote in a three-member district with seven candidates

Droop quota = 100 ballots/(3 + 1) seats + 1 = 26 ballots

Three preferences		Two preferences		One preference	
Number of ballots	**Rankings**	**Number of ballots**	**Rankings**	**Number of ballots**	**Ranking**
15	P-Q-R	20	S-T	17	U
15	P-R-Q	9	T-S	13	V
8	Q-R-P				
3	R-P-Q				

Candidate	First Count	Second Count	Third count	Fourth count	Fifth count	Sixth count
P	30	-4 =26	26	26	26	26
Q	8	+2 =10	+5 =15	15	15	15
R	3	+2 = 5	-5 = 0	0	0	0
S	20	20	20	+9 = 29	-3 =26	26
T	9	9	9	-9 = 0	0	0
U	17	17	17	17	17	17
V	13	13	13	13	13	-13 = 0
Non-transferable	-	-	-	-	+3 = 3	+13 =16

5. An Analysis of Proportional Representation Electoral Systems

5.1　　　　The proportionalists advocate that the function of the electoral system is to produce a representative body that most nearly approximates the patterns of belief of the electorate at large, and any system that frustrates the replication of the electoral mood is perversion of the principle of democratic representation.

5.2　　　　Both the list and the STV systems satisfy the basic aim of the proportionalists. There are few wasted votes, and those are of the small parties not able to gain even close to the quota and probably not able to muster the strength to achieve representation under any electoral system.   It is unlikely for a large party to achieve an inordinate share of the representative body.

5.3　　　　However, there are some inequities in the two systems. Under the list system, it is certainly possible for the total vote of the very small and unrepresented parties to be equal to several quotas, while the total vote of the candidates excluded under the STV might be equal to one or more quotas.

5.4　　　　The systems of PR would reduce markedly the ability of any one party to achieve a majority in the election. In any system of PR, the emphasis is on the replication of social groupings, and inherently this will mean in any but the most cohesive state the representation of more than two parties. Hence, the effect of proportional systems seems that the parties would form coalitions to share an intermediate view.   On this basis, it would be difficult to produce an opposition with a distinct political nucleus.

Concerns of List Proportional Representation

5.5　　　　There is, first, the role of the party, which can almost dictate the election of party functionaries by placing them on the ballot and conversely punish the irritating by placing the candidate far down the ballot. Because the party has the power to reward and punish by the rank on the list ballot, it in effect has the power to restrict the role of candidate to the whims of the party leadership and party machinery, forcing an ideological consistency which might deprive the candidate of the freedom of action to which the candidate might feel entitled.

5.6　　　　The second concern of the list system is the insertion of party between the voter and the representative. The elector cannot express the displeasure through any means but refusing to vote for the party, which seems a drastic move, if only one of the favorites is placed down the list. The list system furnishes the voter with no recourse other than acceptance of the list as it stands at election time.

5.7   Under the list system, parties play an important role. In order to be successful in the election, parties have to publicize their beliefs and differentiate their candidates from the others.   Hence, independent candidates may have to form alliances or are affiliated to parties for the election.   Many new parties would be formed accordingly.

References

1.   Vernon Bogdanor, What is Proportional Representation? A guide to the issues, 1984.

2.   Vernon Bogdanor and David Butler, Democracy and Elections, Electoral Systems and their Political Consequences, 1983.

3.   Ron Gould, Christine Jackson and Loren Wells, Strengthening Democracy: A Parliamentary Perspective, 1995.

4.   Bernard Grofman and Arend Lijphart, Electoral Laws and their Political Consequences, 1986.

5.   Peter Hain, Proportional Misrepresentation: The Case Against PR in Britain, 1986.

6.   Enid Lakeman, How Democracies Vote: A Study of Majority and Proportional Electoral Systems, 1970.

7.   Enid Lakeman, Power to Elect: The Case for Proportional Representation, 1982.

8.   Arend Lijphart, Electoral Systems and Party Systems, 1984.

9.   A. J. Milnor, Elections and Political Stability, 1969.

10.   Gerald M. Pomper, Choosing an Electoral System: Issues and Alternatives, 1994.

11.   Douglas W. Rae, The Political Consequences of Electoral Laws, 1969.

12.   林尚立，選舉政治，1993年。

13.   政制事務局，香港特別行政區，第一屆立法會選舉新增功能界別選民劃分及選舉委員會選民劃分諮詢文件，1997年7月。

14.   雷競璇，選舉制度概論，1985。