資助機構薪酬及附帶福利聯委會

JOINT COUNCIL ON SALARY AND FRINGE BENEFITS IN SUBVENTED ORGANIZATIONS (JCSSO)

Introduction:

It is very kind that the government is now subsidizing 3% of the interest of the property mortgage for employees of subsidized bodies. *They shall be referred as the employees (or the employee) in this letter.* We propose that the subsidy can be changed as a lump sum to help the employees to pay for the initial down payment. We use the Net Present Value approach to find the lump sum the money involved. The key mathematical results are listed, followed by a table which shows the lump sum of typical situations. Finally, the derivation of the mathematical results is attached as an appendix.

The key formulae:

Let us define our symbol first:

the principal a bank lent to the employee P,
the yearly rate R,
the monthly amount (mortgage) to be paid back to the bank A
and the number of years for the mortgage N.

Then the monthly rate is $\frac{R}{12}$, which we shall call it r and the number of months to paid for the mortgage is $12N$, which we shall call it n. These symbols are introduced to simplify the formulae. We have the following relationship:

$$A = rP\left(1 + \frac{1}{(1+r)^n - 1}\right) \quad (1)$$

If the government is going to subsidize the employee for 10 years, then the net present value of the lump sum is given by

$$NPV = \frac{0.3P}{1+r} - \frac{0.3A}{r(1+r)} + \frac{A}{400r^2} \times \left[1 - \frac{1}{(1+r)^{120}}\right] \quad (2)$$

Given, N, the number of years, R, the annual rate, and P the initial debt, the monthly mortgage A can be found by (1) (or simply the bank should tell the employee the value of the monthly mortgage). Provided that the rate subsidy by the government is 3% for 10 years, the lump sum that should be paid to the staff is given by (2)

Typical numerical results:

		Initial Debt(Million)		
		1.0	2.0	3.0
rate	8%	$182635	$365271	$547906
	10%	$172057	$344.114	$516171

Table 1: The lump sum the government required to pay to the employee.

Appendix

The monthly rate is r ($r = \frac{R}{12}$) and the number of months to paid for the mortgage is n ($n = 12N$). We have

$$P(1+r)^n = A(1+r)^{n-1} + A(1+r)^{n-2} + A(1+r)^{n-3} + \cdots A(1+r)^1 + A$$

RHS is a sum of GP.

$$P(1+r)^n = A\frac{(1+r)^n - 1}{r}$$

and the value of A is given by:

$$A = \frac{P(1+r)^n r}{(1+r)^n - 1} = rP\left(1 + \frac{1}{(1+r)^n - 1}\right) \qquad (1)$$

Let $D(n)$, $I(n)$, $PV(n)$ denotes, the debt remain, the interest subsidy provided by the government and the Present value of the subsidy. The following table shows the cash flow:

Number of months, n	Debt, $D(n)$	Interest paid by the government, $I(n) = (D(n-1) \times r) \times \frac{3\%}{R} = \frac{D(n-1)}{400}$	The net present value of the interest paid by the government, $PV(n) = \frac{I(n)}{(1+r)^n}$
0	P	0	
1	$P(1+r) - A$	$\frac{P}{400}$	$\frac{P}{400} \times \frac{1}{1+r}$
2	$P(1+r)^2 - A\frac{(1+r)^2 - 1}{r}$	$\frac{P(1+r) - A}{400}$	$\frac{P}{400} \times \frac{1}{1+r} - \frac{A}{400} \times \frac{1}{(1+r)^2}$
3	$P(1+r)^3 - A\frac{(1+r)^3 - 1}{r}$	$\frac{P(1+r)^2 - A\frac{(1+r)^2 - 1}{r}}{400}$	$\frac{P}{400} \times \frac{1}{1+r} - \frac{A}{400r} \times \left[\frac{1}{1+r} - \frac{1}{(1+r)^3}\right]$
t	$P(1+r)^t - A\frac{(1+r)^t - 1}{r}$	$\frac{P(1+r)^{t-1} - A\frac{(1+r)^{t-1} - 1}{r}}{400}$	$\frac{P}{400} \times \frac{1}{1+r} - \frac{A}{400r} \times \left[\frac{1}{1+r} - \frac{1}{(1+r)^t}\right]$

The debt remains at the t-th month is

$$D(t) = P(1+r)^t - A\frac{(1+r)^t - 1}{(1+r)-1}$$

$$= P(1+r)^t - A\frac{(1+r)^t - 1}{r}$$

The interest paid in the t-th month is $D(t\text{-}1)r$. As the government subsidizes only 3% in rate R, the interest paid by the government is $I(t) = (D(t-1)r)\times\frac{3\%}{R} = \frac{D(t-1)}{400}$. (note that $\frac{r}{R} = \frac{1}{12}$ by definition). The present value of $I(t)$ is $I(t)$ divided by the (discount factor)^t. Thus $PV(t) = \frac{I(t)}{(1+r)^t}$. As the government only supports the staff for 120 months (10 years), we need to sum $PV(1)+PV(2)+PV(3)+\cdots PV(120)$

The total amount the government should give to the staff is:

$$NPV = \sum_{t=1}^{120}\frac{P}{400}\times\frac{1}{1+r} - \frac{A}{400r}\times\left[\frac{1}{1+r} - \frac{1}{(1+r)^t}\right]$$

$$= \frac{0.3P}{1+r} - \frac{0.3A}{r(1+r)} + \frac{A}{400r}\sum_{t=1}^{120}\frac{1}{(1+r)^t}$$

$$= \frac{0.3P}{1+r} - \frac{0.3A}{r(1+r)} + \frac{A}{400r}\times\frac{1}{1+r}\,\frac{1-\frac{1}{(1+r)^{120}}}{1-\frac{1}{1+r}}$$

$$= \frac{0.3P}{1+r} - \frac{0.3A}{r(1+r)} + \frac{A}{400r^2}\times\left[1 - \frac{1}{(1+r)^{120}}\right]$$

$$NPV = \frac{0.3P}{1+r} - \frac{0.3A}{r(1+r)} + \frac{A}{400r^2}\times\left[1 - \frac{1}{(1+r)^{120}}\right] \qquad (2)$$

End of derivation.